Naïve Bayes

CS 780/880 Natural Language Processing Lecture 8

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Last lecture

Key idea: Probabilistic language modeling

Concepts

- Conditional probability
- Chain rule
- N-gram models
- Uses of language models
  - Generation
  - Evaluation
- Perplexity

Unigram model

Basic idea: model the text as the individual words occurring independently

- Parametrized by corpus token frequencies

$$P(w^{(1)} \dots w^{(N)}) = \prod_{i=1}^{N} P(w^{(i)})$$

What's the problem with this?

Bigram model

Basic idea: model text as words being dependent on **only** the prior word

- Parameterized by token co-occurrence frequencies

$$P(w^{(1)} \ldots w^{(N)}) = \prod_{i=1}^{N} P(w^{(i)} \mid w^{(i-1)})$$

A bigram model is a type of **Markov Chain**

Markov Chain

Definition: a discrete stochastic process with the Markov property :

$$P(Xt|Xt-1,\dots,X_1) = P(Xt|Xt-1)$$

fully determined by a probability transition matrix P which defines the transition probabilities:

$$(P_{ij} = P(X_t = j|Xt-1 = i)$$

and an initial probability distribution specified by the vector x where:

$$x_i = P(X_0 = i)$$

Should hopefully be clear why a bigram model is this

In general, we're often concerned about the **stationary distribution** of X_t over time

- Not so much in NLP

https://stephens999.github.io/fiveMinuteStats/markov_chains_discrete_intro.html

Stationary distribution

Corpus:

“i am a person .”

“a person am i .”

Stationary distribution: from the CPT alone, what is the probability that the word W_t occurring at time t will be a particular word *w*?

- If you think of words as states, can also be: what percentage of our total time will we spend in each state?
- Important for some tasks, not really so much for NLP
- Can be solved by doing eigendecomposition on the CPT

CPT:

	i	am	a	person	.	[END]
[START]	0.5	0	0.5	0	0	0
i	0	0.5	0	0	0.5	0
am	0.5	0	0.5	0	0	0
a	0	0	0	1	0	0
person	0	0.5	0	0	0.5	0
.	0	0	0	0	0	1.0

Bayes Rule

Conditional probability

When two variables may be dependent, then their joint probability is expressed as follows:

$$P(X,Y) = P(Y)P(X|Y) = P(X)P(Y|X)$$

If they happen to be independent, then $P(X|Y) = P(X)$ and $P(Y|X) = P(Y)$, so

$$P(X,Y) = P(Y)P(X) = P(X)P(Y)$$

Bayes Rule

It follows from

$$P(X,Y) = P(Y)P(X|Y) = P(X)P(Y|X)$$

that

$$P(X|Y) = \frac{P(Y|X)P(X)}{P(Y)}$$

Examples

$$P(Lung\ cancer|Cough) = \frac{P(Cough|Lung\ cancer)P(Lung\ cancer)}{P(Cough)}$$

$$P(Conspiracy|Event) = \frac{P(Event|Conspiracy)P(Conspiracy)}{P(Event)}$$

$$P(Barista\ likes\ you|Smiles\ when\ they\ give\ you\ coffee)$$

$$= \frac{P(Smiles\ when\ they\ give\ you\ coffee|Barista\ likes\ you)P(Barista\ likes\ you)}{P(Smiles\ when\ they\ give\ you\ coffee)}$$

Relative probabilities

Often we only care about the relative probability of two possible outcomes, rather than their true probability:

$$P(Lung\ cancer|Cough) \text{ vs. } P(COVID|Cough)$$

$$\frac{P(Cough|Lung\ cancer)P(Lung\ cancer)}{P(Cough)} \text{ vs. } \frac{P(Cough|COVID)P(COVID)}{P(Cough)}$$

Because we only care about the relative value, we can ignore the denominator

$P(Cough|Lung\ cancer) \approx P(Cough|COVID) \approx 1.0$

~50 million COVID cases in 2022, ~300k new lung cancer cases in 2023

So P(COVID) = .15, and P(Lung cancer) = 0.001

So $P(COVID|Cough)$ is **150 times** higher than $P(Lung\ cancer|Cough)$

https://www.cancer.org/cancer/lung-cancer/about/key-statistics.html

https://covid.cdc.gov/covid-data-tracker/#trends_totalcases_select_00

Base rate fallacy

A lot of fallacious thinking comes from ignoring the **base rates** P(X) and P(Y) in $\frac{P(Y|X)P(X)}{P(Y)}$

$$P(Hypothesis \mid Rare\ event) = \frac{P(Rare\ event \mid Hypothesis)P(Hypothesis)}{P(Rare\ event)}$$

P(Hypothesis) is often lower than you think
P(Rare event) is often higher than you think

https://en.wikipedia.org/wiki/Base_rate_fallacy
https://en.wikipedia.org/wiki/List_of_cognitive_biases

Naïve Bayes

Application to text

Classification:

$$P(Class \mid Words)$$

$$P(Class\ 0 \mid Words) \text{ vs. } P(Class\ 1 \mid Words)$$

$$\frac{P(Words \mid Class\ 0)P(Class\ 0)}{P(Words)} \text{ vs. } \frac{P(Words \mid Class\ 1)P(Class\ 1)}{P(Words)}$$

We can ignore P(Words), but how do we calculate:

- $P(Words \mid Class\ 0)$
- $P(Class\ 0)$
- $P(Words \mid Class\ 1)$
- $P(Class\ 1)$

Application to text

$$P(Class\ 0) = \frac{\#\ Class\ 0}{\#\ Class\ 0 + \#\ Class\ 1}$$

- And likewise for class 1

$P(Words \mid Class\ 0)$

- Build an n-gram model of all texts for which class is Class 0
- Use this model to estimate $P(Words \mid Class\ 0)$
- And likewise for Class 1

Naïve Bayes

Basic idea: apply Bayes rule to find relative likelihoods of $P(Class\ 0 \mid Words)$ vs. $P(Class\ 1 \mid Words)$, using **unigram model** for $P(Words \mid Class\ C)$

So if we consider words = $\{w_0, w_1, \ldots, wN\}$:

$$P(Class\ 0 \mid Words) \propto P(Class\ 0) \prod_{i=1}^{N} P(wi \mid Class\ 0)$$

$$P(Class\ 1 \mid Words) \propto P(Class\ 1) \prod_{i=1}^{N} P(wi \mid Class\ 1)$$

Read the SST-2 dataset

```
1 display(dev_df)
```

	sentence	label
0	it 's a charming and often affecting journey .	1
1	unflinchingly bleak and desperate	0
2	allows us to hope that nolan is poised to emba...	1
3	the acting , costumes , music , cinematography...	1
4	it 's slow -- very , very slow .	0
...	...	...
867	has all the depth of a wading pool .	0
868	a movie with a real anarchic flair .	1
869	a subject like this should inspire reaction in...	0
870	... is an arthritic attempt at directing by ca...	0
871	looking aristocratic , luminous yet careworn i...	1

872 rows × 2 columns

Preprocess and vectorize the data

```
from nltk import PorterStemmer
```

```
# for this dataset, the tokenization has already been done for us
stemmer = PorterStemmer()
def preprocess(s):
  return ' '.join([stemmer.stem(token) for token in s.strip().split(' ')])
```

```
train_df['preprocessed'] = train_df['sentence'].apply(preprocess)
dev_df['preprocessed'] = dev_df['sentence'].apply(preprocess)
```

```
from sklearn.feature_extraction.text import CountVectorizer
```

```
# Why are we using a CountVectorizer here instead of TF-IDF?

vectorizer = CountVectorizer()
train_X = vectorizer.fit_transform(train_df['preprocessed'])
dev_X = vectorizer.transform(dev_df['preprocessed'])
```

Build and evaluate the model

```
from sklearn.naive_bayes import MultinomialNB
```

```
# See https://scikit-learn.org/stable/modules/generated/sklearn.naive_bayes.MultinomialNB.html#sklearn.naive_bayes.MultinomialNB
# for hyperparameter options

model = MultinomialNB()
```

```
model.fit(train_X, train_df['label'])
```

```
MultinomialNB()
```

```
dev_py = model.predict(dev_X)
```

```
from sklearn.metrics import accuracy_score, precision_score, recall_score, f1_score
```

```
def evaluate_predictions(y, py):
  print(f'Accuracy: {accuracy_score(y, py):.3f}')
  print(f'Precision: {precision_score(y, py):.3f}')
  print(f'Recall: {recall_score(y, py):.3f}')
  print(f'F1: {f1_score(y, py):.3f}')
```

Build and evaluate the model

Naïve Bayes:

```
# Evaluating on the dev set
evaluate_predictions(dev_df['label'], dev_py)
```

```
Accuracy: 0.807
Precision: 0.794
Recall: 0.840
F1: 0.816
```

```
# Evaluating on a sample of the training set
train_py = model.predict(train_X[0:1000])
evaluate_predictions(train_df['label'].iloc[0:1000],train_py)
```

```
Accuracy: 0.891
Precision: 0.894
Recall: 0.906
F1: 0.900
```

K-nearest-neighbors:

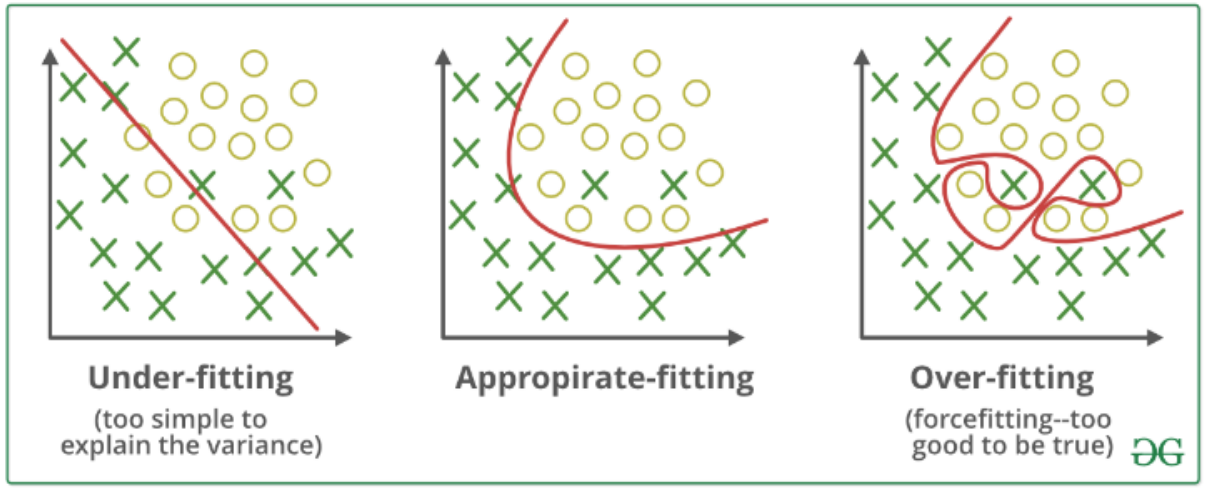
```
evaluate_model(dev_X, dev_y, classifier)
```

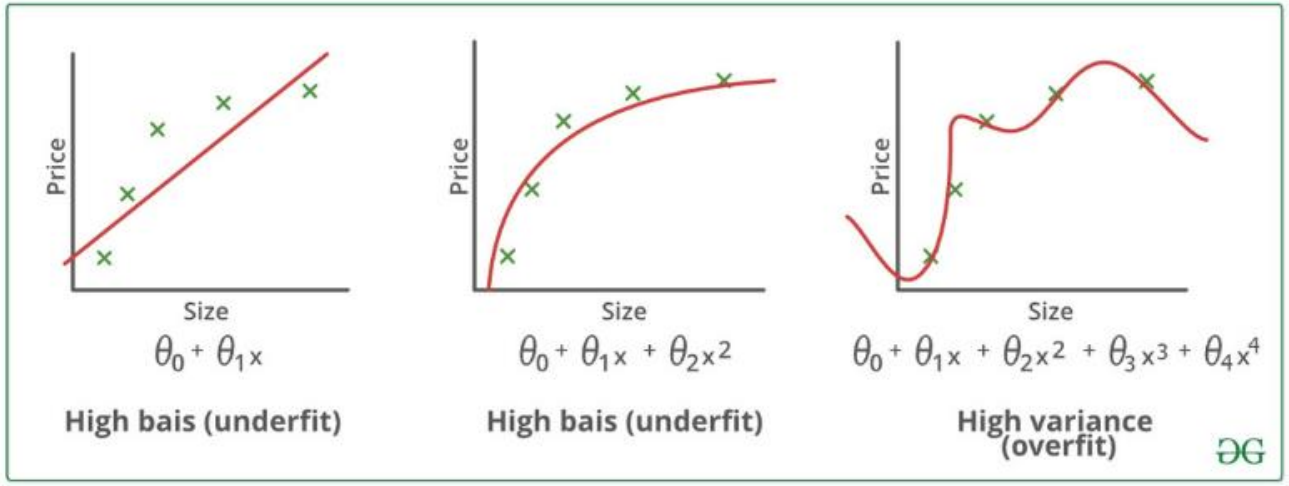
```
Accuracy: 0.742
Precision: 0.707
Recall: 0.842
F1: 0.769
```

```
evaluate_model(train_X[0:1000], train_y[0:1000], classifier)
```

```
Accuracy: 0.948
Precision: 0.945
Recall: 0.959
F1: 0.952
```

Overfitting and underfitting





Overfitting: model overly tuned to quirks of the training data—doesn't generalize

Underfitted: model not tuned enough to training data—doesn't capture data structure

Related (but not identical) to **bias-variance trade-off**

- High bias → underfitting
- High variance → overfitting

https://www.geeksforgeeks.org/underfitting-and-overfitting-in-machine-learning/

Explaining the model

```
1 # We can get the (log) probability of each word for each class
2 print('Word log-probs:')
3 display(model.feature_log_prob_)
4
5 # It will have one row for each class and one column for each word in the vocabulary
6 print('Log-probs matrix shape:')
7 display(model.feature_log_prob_.shape)
8
```

```
Word log-probs:
array([[-11.12495518,  -8.09240893, -10.20866444, ..., -12.51124954,
        -10.71949007, -10.90181162],
       [-11.03503481,  -9.50897851, -10.15956608, ..., -11.25817837,
        -12.64447273, -11.95132555]])
Log-probs matrix shape:
(2, 10106)
```

Explaining the model

```
# We can identify the words that were the biggest distinguishers by calculating
# the diff between the two rows
word_prob_diffs = model.feature_log_prob_[0] - model.feature_log_prob_[1]
word_prob_diffs
```

```
array([-0.08992036,  1.41656958, -0.04909837, ..., -1.25307117,
        1.92498266,  1.04951392])
```

```
# And then we can use numpy.argsort() and numpy.abs() to find the indices of the
# words with the biggest diff (positive or negative)
import numpy as np
sorted_diff_indices = np.argsort(np.abs(word_prob_diffs))
sorted_diff_indices
```

```
array([6103, 9942, 7833, ..., 6692, 6721, 9402])
```

```
# Numpy argsort always goes in ascending order, so to get the top K indices
# we have to grab the last K indices

# We can use -1 as the third part of our slice, to get these back in reverse order
k= 10
top_k_indices = sorted_diff_indices[:-k:-1]
```

Explaining the model

```python
# Then we can find the words and values associated with those indices
vocab = vectorizer.get_feature_names_out()
top_words = vocab[top_k_indices]
top_diffs = word_prob_diffs[top_k_indices]

print(f'Top {k} distinguishing words in our Naive Bayes classifier')
for word, diff in zip(top_words,top_diffs):
  print(f'\tWord:"{word}" - Diff: {diff:.3f}')
```

```
Top 10 distinguishing words in our Naive Bayes classifier
        Word:"unfunni" - Diff: 4.861
        Word:"poorli" - Diff: 4.698
        Word:"pointless" - Diff: 4.677
        Word:"tiresom" - Diff: 4.588
        Word:"eleg" - Diff: -4.410
        Word:"unnecessari" - Diff: 4.410
        Word:"badli" - Diff: 4.382
        Word:"embrac" - Diff: -4.355
        Word:"inept" - Diff: 4.338
```

Interpreting log-probability differences

If :

$$\log(P(w_i|\ class\ 0))\ -\ \log(P(w_i|\ class\ 1))\ =\ 4.8$$

Then:

$$\frac{P(wi|class\ 0)}{P(wi|class\ 1)} = e^{4.8} = 2.718^{4.8} = 121.51$$

Meaning that w_i (“unfunny” in this case) is **121.51** times more likely to occur in class 0 than in class 1

Explaining individual predictions

```
sentence = 'the movie was pretty awful : not good at all .'
preprocessed_sentence = preprocess(sentence)
sentence_x = vectorizer.transform([preprocessed_sentence])
py = model.predict(sentence_x)
print(f'Model prediction for "{preprocessed_sentence}": {py}')
```

```
Model prediction for "the movi wa pretti aw : not good at all .": [0]
```

```
# we can find the vocab indices for the tokens in the sentence
sentence_tokens = preprocessed_sentence.split(' ')
token_indices = [vectorizer.vocabulary_[token] for token in sentence_tokens \
                 if token in vectorizer.vocabulary_] #there's one or two stopwords to ignore
token_indices
```

```
[8892, 5827, 9702, 6828, 680, 6067, 3791, 615, 339]
```

Explaining individual sentences

Is this overfitting?

```
# Then we can do the same thing as we did with the top indices above
sentence_words = vocab[token_indices]
sentence_diffs = word_prob_diffs[token_indices]
print(f'Class probability differences for tokens in the sentence:')
for word, diff in zip(sentence_words,sentence_diffs):
  print(f'\tWord:"{word}" - Diff: {diff:.3f}')
```

```
Class probability differences for tokens in the sentence:
        Word:"the" - Diff: -0.032
        Word:"movi" - Diff: 0.196
        Word:"wa" - Diff: 0.836
        Word:"pretti" - Diff: -0.643
        Word:"aw" - Diff: 1.574
        Word:"not" - Diff: 0.679
        Word:"good" - Diff: -1.021
        Word:"at" - Diff: 0.080
        Word:"all" - Diff: 0.111
```

Concluding thoughts

Naïve bayes: application of Bayes Rule + unigram language modeling to classification

Huge deal in 1998

Limitations?